α-β-ALMOST COMPACTNESS FOR CRISP SUBSETS OFA FUZZY TOPOLOGICAL SPACE

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ABSTRACT :

This paper deals with a new type of compactness, viz., α-β-almost compactness for crisp subsets of a fuzzy topological space X by using the concept of α-shading initiated by Gantner et al. [5]. Several characterizations of such subsets are obtained and also taking ordinary nets and power-set filterbases as basic appliances some characterizations of such subsets have been done.

KEY WORDS : α-β-almost compact space, α-β-adherent point of net and filterbase, α-β-interiorly finite intersection property.

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INTRODUCTION

From very beginning, many mathematicians have engaged themselves to introduce different types of compactness in a fuzzy topological space (henceforth to be abbreviated as fts). In 1978, Gantner, Steinlage and Warren [5] introduced a sort of α-level covering termed as α-shading and using this concept a new type of compactness, viz., α-compactness has been introduced which is a generalization of compactness in an fts. Using the idea of α-shading, in this paper we define the idea of α-β-almost compactness in an fts. A suitably defined adherence of ordinary nets and power-set filterbases, α-β-almost compactness for crisp subsets is also characterized, these characterizations being also true for α-β-almost compactness of X if one puts $A = X$.

PRELIMINARIES

Throughout the paper, by (X, τ) or simply by X, we mean an fts in the sense of Chang [3]. By a crisp subset A of an fts X, we always mean A is an ordinary subset of the set X, the underlying structure of the set X being a fuzzy topology τ, whereas a fuzzy set A [9] in an fts X denotes, as usual, a function from X to the closed interval $I = [0, 1]$ of the real line, i.e., $A \in I^X$. For a fuzzy set A, the closure [3] and interior [3] of A in X will be denoted by $cl\, A$ and $int\, A$ respectively. The support of a fuzzy set A in X will be denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. A fuzzy point [8] in $X$with the singleton support $\{x\} \subseteq X$and the value t (0 <t≤ 1) at x will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking respectively the constant values 0 and 1 on X. The complement of a fuzzy set A in X will be denoted by $1_X \setminus A$ [9], defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A and B in X, we write $A \leq B$ iff $A(x) \leq B(x)$, for each $x \in X$, while we write AqB to mean A is quasi-coincident (q-coincident, for short) with B [8], i.e., if there exists$x \in X$ such that $A(x) + B(x) > 1$; the negation of these two statements are written as$A \nleq B$ and $A\bar{q}B$ respectively. A fuzzy set A in X is called fuzzy regular open [1] if $A = int\, cl\, A$. A fuzzy set B is called a quasi-neighbourhood (q-nbd, for short) [8] of a fuzzy set A if there is a fuzzy open set U in X such that $qU \leq B$.

§ 1. FUZZY β-OPEN AND FUZZY β-CLOSED SETS : SOME RESULTS

Now we recall some definitions, theorem and result for ready references.

DEFINITION 1.1 [4]. A fuzzy set A in an fts X is said to be fuzzy β-open if $A \leq clintclA$. The complement of a fuzzy β-open set is called fuzzy β-closed.

DEFINITION 1.2 [4]. The union of all fuzzy β-open sets in an fts X, each contained in a fuzzy set A in X, is called the fuzzy β-interior of A and is denoted by $\beta intA$.
A fuzzy set A is fuzzy β-open if and only if $A = \beta intA$.

DEFINITION 1.3 [2]. A fuzzy set A in an fts X is called a fuzzy β-open q-nbd of a fuzzy point x_t in X if there exists a fuzzy β-open set V in X such that $x_t qV \leq A$.

DEFINITION 1.4 [4]. The intersection of all fuzzy β-closed sets in an fts X containing the fuzzy set A is called fuzzy β-closure of A, to be denoted by βclA.
A fuzzy set A in an fts X is fuzzy β-closed if and only if $A = \beta clA$.

RESULT 1.5 [2]. A fuzzy point x_t in an fts X belongs to the fuzzy β-closure of a fuzzy set A in X if and only if every fuzzy β-open q-nbd of x_t is q-coincident with A.

THEOREM 1.6 [2]. For any two fuzzy β-open sets A and B in an fts X, A$\bar{q}B \Rightarrow \beta clA\, \bar{q}B$ and $A\bar{q}\beta clB$.

§ 2. α-β-ALMOST COMPACTNESS : CHARACTERIZATIONS

We first recall the definition of α-shading given by Gantner et al. [5]. When this concept is applied to arbitrary crisp subsets of X we get the following definition.

DEFINITION 2.1 [5]. Let A be a crisp subset of an fts X. A collection $\mathscr{U}$ of fuzzy sets in X is called an α-shading (where 0 <α< 1) of A if for each $x \in A$, there is some $U_x \in \mathscr{U}$ such that $U_x(x)$ >α. If, in addition, the members of $\mathscr{U}$ are fuzzy open (β-open) then $\mathscr{U}$ is called a fuzzy open (resp. β-open) α-shading of A.

DEFINITION 2.2. Let X be an fts and A be a crisp subset of X. A is said to be α-compact [5] (resp., α-almost compact [7]) if for every fuzzy open α-shading (0 <α< 1) $\mathscr{U}$ of A, there is a finite (resp., finite proximate) α-subshading of A, i.e., there is a finite subcollection $\mathscr{U}_0$ of $\mathscr{U}$ such that $\{U: U \in \mathscr{U}_0\}$ (resp., $\{cl\, U : U \in \mathscr{U}_0\}$) is again an α-shading of A. If $A = X$ in addition, then X is called an α-compact (resp., α-almost compact) space.

We now set the following definition.

DEFINITION 2.3. Let X be an fts and A, a crisp subset of X. A is called α- β-almost compact if each fuzzy β-open α-shading of A has a finite β-proximate α-subshading, i.e., there exists a finite subcollection $\mathscr{U}_0$ of $\mathscr{U}$ such that $\{\beta clU : U \in \mathscr{U}_0\}$ is again an α-shading of A. If, in addition, $A = X$, then X is called an α- β-almost compact space.

It follows from Definition 2.3 that

THEOREM 2.4.(a) Every finite subset of an fts X is α- β-almost compact.

(b) If A_1 and A_2 are α-β-almost compact subsets of an fts X, then so is $A_1 \cup A_2$.

(c) X is α-β-almost compact if X can be written as the union of finite number of α- β-almost compact sets in X.

As $\beta clA \leq clA$, for any fuzzy set A in an fts X, it is clear from definition that α-β-almost compactness imply α-almost compactness, but not conversely. To achieve the converse we need to define some sort of regularity condition in our setting. The following definition serves our purpose.

DEFINITION 2.5. An fts X is said to be α-β-regular, if for each point $x \in X$ and each fuzzy open set U_x in X with $U_x(x) > \alpha$, there exists a fuzzy β-open set V_x in X with $V_x(x) > \alpha$ such that $\beta cl V_x \leq U_x$.

Two other equivalent ways of defining α- β-regularity are given by the following result.

THEOREM 2.6. For an fts X , the following are equivalent :

(a) X is α- β-regular.

(b) For each point $x \in X$ and each fuzzy closed set F with $F(x) < 1 - \alpha$, there is a fuzzy β-open set U such that $(\beta clU)(x) < 1 - \alpha$ and $F \leq U$.

(c) For each $x \in X$ and each fuzzy closed set F with $F(x) < 1 - \alpha$, there exist fuzzy β-open sets U and V such that $V(x) > \alpha, F \leq U$ and $U\bar{q}V$.

PROOF. (a) ⇒ (b) : Let $x \in X$ and F be a fuzzy closed set with $F(x) < 1 - \alpha$. Put $V = 1_X \backslash F$. Then V is a fuzzy open set and $V(x) > \alpha$. By (a), there is a fuzzy β-open set W in X with $W(x) > \alpha$ and $\beta clW \leq V = 1_X \backslash F$. Then $F \leq 1_X \backslash \beta clW = \beta int\,(1_X \backslash W) = U$ (say). Then U is fuzzy β-open in X. Also, $\beta clU = \beta cl\big(\beta int\,(1_X \backslash W)\big) = \beta cl(1_X \backslash \beta clW) = 1_X \backslash \beta int(\beta clW) \leq 1_X \backslash W$. Thus $(\beta clU)(x) \leq (1_X \backslash W)(x) < 1 - \alpha$.

(b) ⇒ (a) : Let $x \in X$ and U be any fuzzy open set in X with $U(x) > \alpha$. Let $F = 1_X \backslash U$. Then F is a fuzzy closed set in X with $F(x) < 1 - \alpha$. By (b), there is a fuzzy β-open set V such that $(\beta clV)(x) < 1 - \alpha$ and $F \leq V$. So $(1_X \backslash \beta clV)(x) > \alpha$, i.e., $W(x) > \alpha$ where $W = 1_X \backslash \beta clV = \beta int(1_X \backslash V)$ is a fuzzy β-open set in X. Now $\beta clW = \beta cl(1_X \backslash \beta clV) = 1_X \backslash \beta int(\beta clV) \leq 1_X \backslash V \leq 1_X \backslash F = U$. Hence (a) follows.

(b) ⇒ (c) : For a given $x \in X$ and a fuzzy closed set F with $F(x) < 1 - \alpha$, there exists (by (b)) a fuzzy β-open set U such that $(\beta clU)(x) < 1 - \alpha$ and $F \leq U$. Then the fuzzy point $x_{1-\alpha} \notin \beta clU$. Hence by Definition 1.4 and Result 1.5, there is a fuzzy β-open set V in X such that $x_{1-\alpha} qV$ and $V\bar{q}U$, i.e., $V(x) + 1 - \alpha > 1 \Rightarrow V(x) > \alpha$.

(c) ⇒ (b) : Let $x \in X$, and F, a fuzzy closed set in X with $F(x) < 1-\alpha$. By (c), there exist fuzzy β-open sets U and V such that $V(x) > \alpha, F \leq U$ and $U\bar{q}V$. Now $V(x) > \alpha \Rightarrow x_{1-\alpha}qV$. Then as $U\bar{q}V$, by Theorem 1.6, $\beta clU\bar{q}V \Rightarrow (\beta clU)(x) \leq 1-V(x) < 1-\alpha$.

THEOREM 2.7. In an α-β-regular fts X, the α-β-almost compactness of a crisp subset A of X implies its α-compactness (and hence α-almost compactness).

PROOF. Let $\mathscr{U}$ be a fuzzy open α-shading of an α-β-almost compact set A in an α-β-regular fts X. Then for each $a \in A$, there exists $U_a \in \mathscr{U}$ such that $U_a(a) > \alpha$. By α-β-regularity of X, there is a fuzzy β-open set V_a in X with $V_a(a) > \alpha$ such that $\beta clV_a \leq U_a$ … (1).
Let $\mathscr{V} = \{V_a : a \in A\}$. Then $\mathscr{V}$ is a fuzzy β-open α-shading of A. By α-β-almost compactness of A, there is a finite subset A_0 of A such that $\mathscr{V}_0 = \{\beta clV_a : a \in A_0\}$ is an α-shading of A. By (1), $\mathscr{U}_0 = \{U_a : a \in A_0\}$ is then a finite α-subshading of $\mathscr{U}$. Hence A is α-compact (and hence α-almost compact).

In what follows in the rest of this paper we would like to give different subsets of X, where X is endowed, as, usual, with a fuzzy topology τ.

Mashhour et al. [6] defined a fuzzy set A in an fts X to be fuzzy regular semiopen if there is a fuzzy regular open set U such that $U \leq A \leq clU$.

REMARK 2.8. It is obvious that fuzzy regular semiopen set is fuzzy β-open. Indeed, A is fuzzy regular semiopen ⇒ there exists a fuzzy regular open set U in X such that $U \leq A \leq clU = clintclU \leq clintclA \Rightarrow A \leq clintclA$.

LEMMA 2.9. If V be a fuzzy β-open set, then $intclV$ is fuzzy regular open.

PROOF. $intclV \leq intcl(clintclV)$ (as V is fuzzy β-open in X) $= intcl(intclV)$. Again $intcl(intclV) \leq intclV$.
Therefore, $intcl(intclV) = intclV$. Hence $intclV$ is fuzzy regular open.

THEOREM 2.10. A subset A of X is α- β-almost compact if and only if every α-shading of A by fuzzy regular semiopen sets in X has a finite β-proximte α-subshading.

PROOF. The proof follows from the definition of α- β-almost compactness and by the Lemma 2.9 which states that whenever $\{V_i : i \in \Lambda\}$ is a fuzzy β-open α-shading of A, then $\{(intclV_i) \cup V_i : i \in \Lambda\}$ is also an α-shading of A by fuzzy regular semiopen sets.

THEOREM 2.11. A crisp subset A of an fts X is α-β-almost compact iff every family of fuzzy β-open sets, the β-interiors of whose β-closures form an α-shading of A, contains a finite subfamily, the β-closures of whose members form an α-shading of A.

PROOF. It is sufficient to observe that for a fuzzy β-open set U, $U \leq \beta \text{int}(\beta clU) \leq \beta cl(\beta int(\beta clU)) = \beta clU$ (Indeed, $\beta clU \leq \beta cl(\beta intU) \leq \beta cl(\beta int(\beta clU))$).

THEOREM 2.12. A crisp subset A of an fts X is α-β-almost compact iff for every collection $\{F_i : i \in \Lambda\}$ of fuzzy β-open sets with the property that for each finite subset Λ_0 of Λ, there is $x \in A$ such that $\inf_{i\in\Lambda_0} F_i(x) \geq 1-\alpha$, one has $\inf_{i\in\Lambda}(\beta cl\, F_i)(y) \geq 1-\alpha$, for some $y \in A$.

PROOF. Let A be α-β-almost compact, and if possible, let for a collection $\{F_i : i \in \Lambda\}$ of fuzzy β-open sets in X with the stated property, $(\bigcap_{i\in\Lambda} \beta cl\, F_i)(x) < 1-\alpha$, for each $x \in A$. Then $\alpha < (1_X \backslash \bigcap_{i\in\Lambda} \beta cl\, F_i)(x) = [\bigcup_{i\in\Lambda}(1_X \backslash \beta cl\, F_i)](x)$, for each $x \in A$ which shows that $\{1_X \backslash \beta cl\, F_i : i \in \Lambda\}$ is a fuzzy β-open α-shading of A. By α-β-almost compactness of A, there is a finite subset Λ_0 of Λ such that $\{\beta cl\,(1_X \backslash \beta cl\, F_i) : i \in \Lambda_0\} = \{1_X \backslash \beta int(\beta cl F_i) : i \in \Lambda_0\}$ is an α-shading of A. Hence $\alpha < [\bigcup_{i\in\Lambda_0}(1_X \backslash \beta int(\beta cl\, F_i))](x) = [1_X \backslash (\bigcap_{i\in\Lambda_0} \beta int(\beta cl\, F_i))](x)$, for each $x \in A$. Then $(\bigcap_{i\in\Lambda_0} F_i)(x) \leq [\bigcap_{i\in\Lambda_0} \beta int(\beta cl\, F_i)](x) < 1-\alpha$, for each $x \in A$, which contradicts the stated property of the collection $\{F_i : i \in \Lambda\}$.

Conversely, let under the given hypothesis, A be not α-β-almost compact. Then there is a fuzzy β-open α-shading $\mathscr{U} = \{U_i : i \in \Lambda\}$ of A having no finite β-proximate α-subshading, i.e., for every finite subset Λ_0 of Λ, $\{\beta cl\, U_i : i \in \Lambda_0\}$ is not an α-shading of A, i.e., there exists $x \in A$ such that $\sup_{i\in\Lambda_0}(\beta cl\, U_i)(x) \le \alpha$, i.e., $1 - \sup_{i\in\Lambda_0}(\beta cl\, U_i)(x) = \inf_{i\in\Lambda_0}(1_X \setminus \beta cl U_i)(x) \ge 1-\alpha$. Hence $\{1_X \setminus \beta cl\, U_i : i \in \Lambda\}$ is a family of fuzzy β-open sets with the stated property. Consequently, there is some $y \in A$ such that $\inf_{i\in\Lambda}[\beta cl\,(1_X \setminus \beta cl\, U_i)](y) \ge 1-\alpha$. Then $\sup_{i\in\Lambda} U_i(y) \le \sup_{i\in\Lambda}[\beta int(\beta cl\, U_i)](y) = 1 - \inf_{i\in\Lambda}[1_X \setminus \beta int\,(\beta cl\, U_i)](y) = 1 - \inf_{i\in\Lambda}[\beta cl\,(1_X \setminus \beta cl U_i)](y) \le \alpha$. This shows that $\{U_i : i \in \Lambda\}$ fails to be an α-shading of A, a contradiction.

§ 3. CHARACTERIZATIONS OF α-β-ALMOST COMPACTNESS VIA ORDINARY NETS AND POWER-SET FILTERBASES

In this section, we characterize α-β-almost compactness of a crisp subset A of an fts X via α-β-adherent point of ordinary nets and power-set filterbases.

Let us now introduce the following definition :

DEFINITION 3.1. Let $\{S_n : n \in (D, \ge)\}$ (where $(D, \ge)$ is a directed set) be an ordinary net in A and $\mathscr{F}$ be a power-set filterbase on A, and $x \in X$ be any crisp point. Then x is called an α-β-adherent point of

(a) the net $\{S_n\}$ if for each fuzzy β-open set U in X with $U(x) > \alpha$ and for each $m \in D$, there exists $k \in D$ such that $k \ge m$ in D and $(\beta cl\, U)(S_k) > \alpha$,

(b) the filterbase $\mathscr{F}$ if for each fuzzy β-open set U with $U(x) > \alpha$ and for each $F \in \mathscr{F}$, there exists a crisp point x_F in F such that $(\beta cl\, U)(x_F) > \alpha$.

THEOREM 3.2. A crisp subset A of an fts X is α-β-almost compact if and only if every net in A has an α-β-adherent point in A.

PROOF. Suppose A is α-β-almost compact, but there is a net $\{S_n : n \in (D, \geq)\}$ in A ($(D, \geq)$ being a directed set, as usual) having no α-β-adherent point in A. Then for each $x \in A$, there is a fuzzy β-open set U_x in X with $U_x(x) > \alpha$, and an $m_x \in D$ such that $(\beta cl\, U_x)(S_n) \leq \alpha$, for all $n \geq m_x$ ($n \in D$). Now, $\mathscr{U} = \{1_X \setminus \beta cl\, U_x : x \in A\}$ is a collection of fuzzy β-open sets such that for any finite subcollection $\{1_X \setminus \beta cl U_{x_i} : i = 1, 2, \ldots, k\}$ (say) of $\mathscr{U}$, there exists $m \in D$ with $m \geq m_{x_i}$, $i = 1, 2, \ldots, k$ in D such that $(\bigcup_{i=1}^{k} \beta cl\, U_{x_i})(S_n) \leq \alpha$, for all $n \geq m$ ($n \in D$), i.e., $\inf_{1 \leq i \leq k} (1_X \setminus \beta cl\, U_{x_i})(S_n) \geq 1 - \alpha$, for all $n \geq m$. Hence by Theorem 2.12, there exists some $y \in A$ such that $\inf_{x \in A} [\beta cl\, (1_X \setminus \beta cl\, U_x)](y) \geq 1 - \alpha$, i.e., $(\bigcup_{x \in A} U_x)(y) \leq [\bigcup_{x \in A} \beta int\, (\beta cl\, U_x)](y) = 1 - [1 - (\bigcup_{x \in A} (\beta int\, (\beta cl\, U_x))(y)] = 1 - \inf_{x \in A} [\beta cl\, (1_X \setminus \beta cl\, U_x)](y) \leq 1 - 1 + \alpha = \alpha$. We have, in particular, $U_y(y) \leq \alpha$, contradicting the definition of U_y. Hence the result is proved.

Conversely, let every net in A have an α-β-adherent point in A and suppose $\{F_i : i \in \Lambda\}$ be an arbitrary collection of fuzzy β-open sets in X. Let Λ_f denote the collection of all finite subsets of Λ, then $(\Lambda_f, \geq)$ is a directed set, where for $\mu, \lambda \in \Lambda_f$, $\mu \geq \lambda$ iff $\mu \supseteq \lambda$. For each $\mu \in \Lambda_f$, put $F_\mu = \cap\{F_i : i \in \mu\}$. Let for each $\mu \in \Lambda_f$, there be a point $x_\mu \in A$ such that $\inf_{i \in \mu} F_i(x_\mu) \geq 1 - \alpha$ …(1).

Then by Theorem 2.12, it is enough to show that $\inf_{i \in \Lambda} (\beta cl\, F_i)(z) \geq 1 - \alpha$ for some $z \in A$. If possible, let $\inf_{i \in \Lambda} (\beta cl\, F_i)(z) < 1 - \alpha$, for each $z \in A$ …(2).

Now, $S = \{x_\mu : \mu \in (\Lambda_f, \geq)\}$ is clearly a net of points in A. By hypothesis, there is an α-β-adherent point z in A of this net. By (2), $\inf_{i \in \Lambda} (\beta cl\, F_i)(z) < 1 - \alpha \Rightarrow$ there exists $i_0 \in \Lambda$ such that $(\beta cl\, F_{i_0})(z) < 1 - \alpha$, i.e., $(1_X \setminus \beta cl\, F_{i_0})(z) > \alpha$. Since z is an α-β-adherent point of S, for the index $\{i_0\} \in \Lambda_f$, there is $\mu_0 \in \Lambda_f$ with $\mu_0 \geq \{i_0\}$ (i.e., $i_0 \in \mu_0$) such that $\beta cl\, (1_X \setminus \beta cl\, F_{i_0})(x_{\mu_0})$

>α, i.e., $\beta int\, \beta cl\, F_{i_0}(x_{\mu_0}) < 1 -$ α. Since $i_0 \in \mu_0$, $\inf_{i \in \mu_0} F_i(x_{\mu_0}) \le F_{i_0}(x_{\mu_0}) \le \beta int\, \beta cl\, F_{i_0}(x_{\mu_0}) < 1 -$ α, which contradicts (1). This completes the proof.

THEOREM 3.3. A crisp subset A of an fts X is α-β-almost compact if and only if every filterbase $\mathscr{F}$ on A has an α-β-adherent point in A.

PROOF. Let A be α-βalmost compact and let there exist, if possible, a filterbase $\mathscr{F}$ on A having no α-β-adherent point in A. Then for each $x \in A$, there exist a fuzzy β-open set U_x with $U_x(x)$ >α, and an $F_x \in \mathscr{F}$ such that $(\beta cl\, U_x)(y) \le$α, for each $y \in F_x$. Then $\mathscr{U} = \{U_x : x \in A\}$ is a fuzzy β-open α-shading of A. By α-β-almost compactness of A, there are finitely many points $x_1, x_2, \ldots, x_n$ in A such that $\mathscr{U}_0 = \{\beta cl\, U_{x_i} : i = 1, 2, \ldots, n\}$ is also an α-shading of A. Choose $F \in \mathscr{F}$ such that $F \le \bigcap_{i=1}^{n} F_{x_i}$. Then $(\beta cl\, U_{x_i})(y) \le$α, for all $y \in F$ and for i = 1, 2, . . . , n. Thus $\mathscr{U}_0$ fails to be an α-shading of A, a contradiction.

Conversely, let the condition hold and suppose, if possible, $\{y_n : n \in (D, \ge)\}$ be a net in A having no α-β-adherent point in A ($(D, \ge)$ being a directed set, as usual). Then for $x \in A$, there are a fuzzy β-open set U_x with $U_x(x)$ >α and an $m_x \in D$ such that $(\beta cl\, U_x)(y_n) \le$α, for all $n \ge m_x$ ($n \in D$). Thus $\mathscr{B} = \{F_x : x \in A\}$, where $F_x = \{y_n : n \ge m_x\}$ generates a filterbase $\mathscr{F}$ on A. By hypothesis, $\mathscr{F}$ has an α-β-adherent point z (say) in A. But there are a fuzzy β-open set U_z with $U_z(z)$ >α and an $m_z \in D$ such that $(\beta cl\, U_z)(y_n) \le$α, for all $n \ge m_z$, i.e., for all $p \in F_z \in \mathscr{B}$ ($\subseteq \mathscr{F}$), $(\beta cl\, U_z)(p) \le$α. Hence z cannot be an α-β-adherent point of the filterbase $\mathscr{F}$, a contradiction. Hence by Theorem 3.2, A is α-β-almost compact.

DEFINITION 3.4. A family $\{F_i : i \in \Lambda\}$ of fuzzy sets in an fts X is said to have α-β-interiorly finite intersection property or simply α-β-IFIP in a subset A of X, if for each finite subset Λ_0 of Λ, there exists $x \in A$ such that $[\bigcap_{i \in \Lambda_0} \beta int\, F_i](x) \ge 1 -$ α.

THEOREM 3.5. A crisp subset A of an fts X is α-β-almost compact if and only if for every family $\mathscr{F} = \{F_i : i \in \Lambda\}$ of fuzzy β-closed sets in X with α-β-IFIP in A, there exists $x \in A$ such that $\inf_{i\in\Lambda} F_i(x) \geq 1 - \alpha$.

PROOF. Assuming A $(\subseteq X)$ to be α-β-almost compact, let $\mathscr{F} = \{F_i : i \in \Lambda\}$ be a family of fuzzy β-closed sets with α-β-IFIP in A. If possible, let for each $x \in A$, $\inf_{i\in\Lambda} F_i(x) < 1 - \alpha$, i.e., $(\bigcap_{i\in\Lambda} F_i)(x) < 1-\alpha$ i.e., $1 - (\bigcap_{i\in\Lambda} F_i)(x) > \alpha \Rightarrow [\bigcup_{i\in\Lambda}(1_X \setminus F_i)](x) > \alpha$. Thus, $\mathscr{U} = \{1_X \setminus F_i : i \in \Lambda\}$ is a fuzzy β-open α-shading of A. By α-β-almost compactness of A, there is a finite subset Λ_0 of Λ such that $[\bigcup_{i\in\Lambda_0} \beta cl\,(1_X \setminus F_i)](x) = 1 - (\bigcap_{i\in\Lambda_0} \beta \operatorname{int} F_i)(x) > \alpha$, i.e., $(\bigcap_{i\in\Lambda_0} \beta int\, F_i)(x) < 1-\alpha$, for each $x \in A$, which shows that $\mathscr{F}$ does not have α-β-IFIP in A, a contradiction.

Conversely, let $\mathscr{U} = \{U_i : i \in \Lambda\}$ be a fuzzy β-open α-shading of A. Thus $\mathscr{F} = \{1_X \setminus U_i : i \in \Lambda\}$ is a family of fuzzy β-closed sets in X with $\inf_{i\in\Lambda} (1_X \setminus U_i)(x) < 1-\alpha$, for each $x \in A$, so that $\mathscr{F}$ does not have α-β-IFIP in A. Hence for some finite subset Λ_0 of Λ, we have for each $x \in A$, $[\bigcap_{i\in\Lambda_0} \beta int(1_X \setminus U_i)](x) < 1 - \alpha \Rightarrow 1 - (\bigcup_{i\in\Lambda_0} \beta cl\, U_i)(x) < 1 - \alpha$, for each $x \in A \Rightarrow$ $(\bigcup_{i\in\Lambda_0} \beta cl\, U_i)(x) > \alpha$, for each $x \in A \Rightarrow A$ is α-β-almost compact.

Putting $A = X$ in the characterization theorems so far of α-β-almost compact crisp subset A, we obtain as follows.

THEOREM 3.6. For an fts (X, τ), the following are equivalent :

(a) X is α-β-almost compact.

(b) Every α-shading of X by fuzzy regular semiopen sets has a finite β-proximate α-subshading.

(c) Every family of fuzzy β-open sets, the β-interiors of whose β-closures form an α-shading of X, contain a finite subfamily, the β-closures of whose members form as α-shading of X.

(d) For every collection $\{F_i : i \in \Lambda\}$ of fuzzy β-open sets in X with the property that for each finite subset Λ_0 of Λ, there is $x \in X$ such that $\inf_{i\in\Lambda_0} F_i(x) \geq 1-\alpha$, one has $\inf_{i\in\Lambda}(\beta cl\ F_i)(y) \geq 1-\alpha$, for some $y \in X$.

(e) Every net in X has an α-β-adherent point in X.

(f) Every filterbase on X has an α-β-adherent point in X.

(g) For every family $\mathscr{F} = \{F_i : i \in \Lambda\}$ of fuzzy β-closed sets in X with α- β-IFIP in X, there exists $x \in X$ such that $\inf_{i\in\Lambda} F_i(x) \geq 1-\alpha$.

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